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Cosmic Entropy from the Bosonic String?

Valerio Faraoni[1,2] and Edgard Gunzig[2,3]

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In Kaluza–Klein models, the compactification of a high number of extra spatial dimensions generates entropy in the observable four-dimensional universe. A Kaluza–Klein cosmological model recently derived from the bosonic string theory in the limit of an infinite number of extra dimensions is compared with the available data from the observations of cosmic microwave background anisotropies.

A possible mechanism to generate a large amount of entropy in the observable universe is the dimensional reduction of a large number d_1 of extra spatial dimensions in Kaluza–Klein (KK) cosmologies; this mechanism has been advocated by many authors, and $d_1 \geq 36$–40 is required for this purpose [1–5].

The study of KK multidimensional cosmology, in which the four observable dimensions expand and the extra dimensions shrink or stay small during the evolution of the universe, has been the subject of a large amount of literature in recent years (see, e.g., refs. [6–15] and references therein). In particular, cosmological inflation in KK models has been investigated by identifying the inflation field with the dilaton, which arises naturally from the geometry, instead of being introduced *ad hoc*. The motivation for these studies is that KK theories are believed to mimic supergravity, superstring, M-theories; superstring, supergravity, and other theories with an extended gravitational sector share the idea of compactified extra (spacelike) dimensions with the simpler KK models. The latter, however, are severely constrained by the observation of the cosmic microwave background, although not all scenarios are ruled out [16]. There is hope that a direct derivation of

[1] Inter-University Centre for Astronomy and Astrophysics, Ganeshkhind, Pune 411 007, India.
[2] RGGR, Faculté des Sciences, Université Libre de Bruxelles, 1050 Bruxelles, Belgium.
[3] Institutes Internationaux de Chimie et Physique Solvay, 1050 Bruxelles, Belgium.

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KK cosmologies from a more fundamental high-energy theory will provide a rigorous framework for KK cosmology, and will modify the values of the KK parameters entering the scalar field potential in such a way that the current observational constraints will be circumvented. However, this goal has not been easy to achieve, until the recent result that the multidimensional cosmologies studied in refs. 6–15 can be obtained from the bosonic string theory in the limit of an infinite number of extra dimensions [17a, 17b]. The starting point is the tree-level effective action [18],[4]

$$S_{\rm eff} = -\frac{1}{2k_0^2}\int d^{D_0}x\sqrt{|g^{(0)}|}\exp(-2\Phi)[R[g^{(0)}] + 4g^{(0)\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + C] \tag{1}$$

where k_0^2 is the D_0-dimensional gravitational constant, Φ is the string dilaton, and C is the central charge deficit. After performing the compactification of d_1 extra dimensions and a conformal transformation to the Einstein frame, the following cosmological effective action is obtained [17a, 17b]:

$$S = -\frac{1}{2k_0^2}\int d^{D_0}x\sqrt{|\hat{g}^{(0)}|}\,\{\hat{R}[\hat{g}^{(0)}] + \hat{g}^{(0)\mu\nu}\partial_\mu\hat{\varphi}\partial_\nu\hat{\varphi} + R[g^{(1)}]\exp(-2\lambda_{(1)}\hat{\varphi}) \tag{2}$$

$$+ 2\Lambda\exp(-2\lambda_{(2)}\hat{\varphi})\} \tag{3}$$

where $\hat{\varphi}$ is the Einstein frame string dilaton, Λ is the cosmological constant in $D_0 + d_1$ dimensions, and $R[g^{(1)}]$ is the Ricci curvature of the manifold of extra dimensions. The cosmological dilatonic coupling constants are given by

$$\lambda_{(1)}^2 = \frac{D_0 + d_1 - 2}{d_1(D_0 - 2)} \tag{4}$$

$$\lambda_{(2)}^2 = \frac{d_1}{(D_0 + d_1 - 2)(D_0 - 2)} \tag{5}$$

as functions of the number of observable and extra dimensions (D_0 and d_1, respectively). The value of the string dilatonic coupling constant

$$\lambda_s = \frac{1}{\sqrt{D_0 - 2}} \tag{6}$$

is reproduced in the limit $d_1 \to \infty$. This limit is not unphysical: a large number of extra dimensions ($d_1 \geq 36$–40) has been invoked by many authors in order to generate a large amount of entropy in the observable 4-dimensional universe [1–5]. It is sometimes remarked (e.g., refs. 17a and 17b) that observa-

[4] The metric signature is $-+++$. The speed of light assumes the value unity, and Newton's constant is related to the Planck mass by $m_{pl} = G^{-1/2}$. Quantities in the Einstein frame are denoted by a caret. The Ricci scalar in the present paper is the opposite of that in ref. 17.

tional cosmology can provide a natural test for string theories—this is indeed the case for the KK cosmology obtained from the $d_1 \to \infty$ limit of the bosonic string. A typical feature of KK cosmology is the dominance of the dilaton field on the dynamics of the early universe; in the model under consideration, it corresponds to the identification of the string dilaton with the inflaton. It is currently believed that inflation is the only causal mechanism capable of solving the problems of the standard big bang model and of successfully generating density perturbations [19a, 19b]. If we adopt this point of view (which is unjustified if an alternative to inflation is found to solve the problems of the standard big bang model), then a test for the string-inspired KK cosmology of refs. 17a and 17b is to determine whether it provides a successful inflationary phase of the universe. To this end, it is necessary to rewrite the action (2) in the canonical form used in the literature on inflation.[5] The redefinition

$$\hat{\varphi} = k_0 \sigma, \qquad k_0 = \frac{\sqrt{8\pi}}{m_{pl}} \tag{7}$$

yields

$$S = -\int d^4x \sqrt{|\hat{g}^{(0)}|} \left[\hat{R}[\hat{g}^{(0)}] \frac{m_{pl}^2}{16\pi} + \frac{1}{2} \hat{g}^{(0)\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + V(\sigma) \right] \tag{8}$$

where

$$V(\sigma) = \frac{R[g^{(1)}]}{2k_0^2} \exp\left(-\alpha \frac{\sigma}{m_{pl}} \right) + \frac{\Lambda}{k_0^2} \exp\left(-\beta \frac{\sigma}{m_{pl}} \right) \tag{9}$$

$$\alpha = 4\sqrt{2\pi\lambda_{(1)}}, \qquad \beta = 4\sqrt{2\pi\lambda_{(2)}} \tag{10}$$

and where the value $D_0 = 4$ has been used. In the limit $d_1 \to \infty$, in which $\lambda_{(1,2)}$ reproduce the string coupling constant (6), the potential (9) reduces to the single exponential

$$V(\sigma) = V_0 \exp\left(-\sqrt{16\pi} \frac{\sigma}{m_{pl}} \right) \tag{11}$$

[5] Unfortunately, the use of an incorrect form of the action seems to be common in the literature on KK theories [16] (see also footnote 11 of ref. 20). Although the error may appear to be insignificant, eventually it has an important effect on the viability of the KK model under consideration [16].

where

$$V_0 = k_0^{-2}\left(\Lambda + \frac{R[g^{(1)}]}{2}\right) \tag{12}$$

This potential is well known from the power-law inflationary scenario [21a, 21b] and, when inserted into the field equations, yields a coasting universe with scale factor $a(t) = a_0 t$ (where a_0 is a constant), which is noninflationary. The power-law inflationary potential $U(\sigma) = U_0 \exp(-\sqrt{16\pi/p}\sigma/m_{pl})$ gives a scale factor $a = a_0 t^p$ and a spectral index of density perturbations $n = 1 - 2/p$. The combined statistical analysis of the COBE and Tenerife observations of cosmic microwave background anisotropies [22] sets the constraint $p \geq 20$ (using the 1σ limit from both experiments). On the basis of the necessity of an inflationary era to explain the observed universe (a point of view that may be questionable for some authors), we conclude that the KK cosmology derived from the bosonic string in ref. 17a and 17b is not viable. The reason may be that the limit $d_1 \to \infty$ of the bosonic string action does not correspond to the real universe, or that the early universe was not dominated by the string dilaton, or perhaps the action (1) is not the correct one. The multidimensional cosmological models considered in ref. 17a and 17b corresponding to finite values of the parameter d_1 (not derived from string theory) were compared with the observational data in ref. 16, and the viable models still lack a rigorous derivation from a high-energy theory. Future research will focus on this topic.

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